Contributed Review: Source-localization algorithms and applications using time of arrival and time difference of arrival measurements

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Locating the position of fixed or mobile sources (i.e., transmitters) based on measurements obtained from sensors (i.e., receivers) is an important research area that is attracting much interest. In this paper, we review several representative localization algorithms that use time of arrivals (TOAs) and time difference of arrivals (TDOAs) to achieve high signal source position estimation accuracy when a transmitter is in the line-of-sight of a receiver. Circular (TOA) and hyperbolic (TDOA) position estimation approaches both use nonlinear equations that relate the known locations of receivers and unknown locations of transmitters. Estimation of the location of transmitters using the standard nonlinear equations may not be very accurate because of receiver location errors, receiver measurement errors, and computational efficiency challenges that result in high computational burdens. Least squares and maximum likelihood based algorithms have become the most popular computational approaches to transmitter location estimation. In this paper, we summarize the computational characteristics and position estimation accuracies of various positioning algorithms. By improving methods for estimating the time-of-arrival of transmissions at receivers and transmitter location estimation algorithms, transmitter location estimation may be applied across a range of applications and technologies such as radar, sonar, the Global Positioning System, wireless sensor networks, underwater animal tracking, mobile communications, and multimedia. © 2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). [http://dx.doi.org/10.1063/1.4947001]

I. INTRODUCTION

Estimation of the location of the source of a signal (e.g., emitting radio, acoustic or optic signals, etc.) has been a subject of research for decades and continues to receive much interest in the signal processing research community, including radar (Bahl and Padmanabhan, 2000 and Yan et al., 2007), sonar (Carter, 1981; Leonard and Durrant-Whyte, 1991; and Leonard and Durrant-Whyte, 2012), mobile communications (Caffery, 2000; Gustafsson and Gunnarsson, 2005; and Caffery and Stuber, 1998), multimedia (Brandstein and Silverman, 1997; Wang and Chu, 1997; and Akyildiz et al., 2007), animal tracking (Spiesberger and Fristrup, 1990), wireless sensor networks (WSNs; Akyildiz et al., 2002; Chen et al., 2002; Patwari et al., 2005; and Mao et al., 2007), and the Global Positioning System (GPS; Hofmann-Wellenhof et al., 2013). However, uncertainties resulting from the modulation of a signal propagated through an inhomogeneous medium and in measurements of received signals can quickly degrade position estimation accuracy when using standard methods to estimate transmitter position (Sayed et al., 2005; Güvenç and Chong, 2009; and Tan et al., 2011). For reference, the acronyms commonly used in the study of transmitter position estimation and adopted by this paper are listed in Table I.

Historically, measurement of the features of received signals needed for input to position estimation algorithms has relied on four methods: time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), and received signal strength (RSS). In recent years, hybrid measurements have been investigated to improve the qualities of measurements of received signals (Mao et al., 2007). Table II provides a comparison of different approaches to obtain metrics for receipt of a signal at a receiver (Güvenç and Chong, 2009 and So, 2011). In this paper, we review localization algorithms and applications of transmitter position estimation models that use TOA and TDOA approaches. These approaches utilize distance-related information between a source (transmitter) and sensors (receivers).

The variance of time-delay measurement errors is inversely proportional to the signal-to-noise ratio (SNR), provided that bandwidth, observation time, and center frequency remain constant (Quazi, 1981). For instance, for high SNR, the standard deviation of the time delay estimate is

\[ \sigma_{\text{TOA}}^2 = \frac{c}{N \cdot \text{SNR} \cdot F(f_0, W)} \],

(1)

where \( N \) is the observation time, \( f_0 \) is the center frequency, \( W \) is the bandwidth, and \( c \) is some constant. High accuracy can be achieved by utilizing high-precision time of arrival measurement techniques at reasonable SNR levels. If TOA or TDOA approaches are used to estimate transmitter location, at least three sensors are required for two-dimensional (2D) position estimation and four are required for three-dimensional (3D) position estimation, according to the principles of trilateration. When more sensors are available, an over-determined system...
TABLE I. List of acronyms and abbreviations.

<table>
<thead>
<tr>
<th>Acronyms and abbreviations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle of arrival</td>
</tr>
<tr>
<td>CRLB</td>
<td>Cramér-Rao lower bound</td>
</tr>
<tr>
<td>FDOA</td>
<td>Frequency difference of arrival</td>
</tr>
<tr>
<td>GPS</td>
<td>Global positioning system</td>
</tr>
<tr>
<td>LOP</td>
<td>Line of position</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-sight</td>
</tr>
<tr>
<td>LS</td>
<td>Least squares</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum likelihood</td>
</tr>
<tr>
<td>MRS</td>
<td>Minimal required number of sensors</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non line-of-sight</td>
</tr>
<tr>
<td>ODS</td>
<td>Over-determined system</td>
</tr>
<tr>
<td>RSS</td>
<td>Received signal strength</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time difference of arrival</td>
</tr>
<tr>
<td>TOA</td>
<td>Time of arrival</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless sensor networks</td>
</tr>
</tbody>
</table>

(ODS) may be solved by using the multilateration method (Boukerche et al., 2007). In most cases, a basic assumption is that there is line-of-sight (LOS) between a transmitter and receiver and the transmitter’s signal follows a direct line-of-sight path to a receiver.

However, in some situations (e.g., a dense urban environment), there is no direct path between a signal source and a sensor because of reflection and diffraction of the signal. In these cases of non-line-of-sight (NLOS) signal propagation, the transmitter location estimate can become significantly biased. To deal with NLOS signal propagation and associated transmitter position estimation error, strategies for NLOS identification and LOS reconstruction have been suggested (Wylie and Holtzman, 1996). NLOS propagation is also a common problem in indoor environments where several limitations are commonly encountered: (1) LOS is not possible, (2) strong multipath fading occurs, (3) GPS does not work well, and (4) it is expensive to deploy specialized infrastructure. Transmitter position estimation methods using TOA, TDOA, or AOA often provide erroneous results because of the first two limitations. Therefore, location fingerprinting approaches are the most promising approach for indoor applications (Prasithsangaree et al., 2002). Under LOS conditions, multipath fading can be identified by the strong peak of the received signal. Cost is another important consideration for transmitter localization systems (Bulusu et al., 2000; Stoleru et al., 2005; and Guo et al., 2011). High-accuracy systems normally require sensor array design optimization, expensive infrastructure, supporting networks, expert installation, and routine maintenance.

In the remainder of this paper, we will present a literature review of TOA or TDOA based source-localization algorithms, including a comparison of the methodologies, advantages, and disadvantages of the various methods (Section II), analysis of the application specific performance of algorithms (Section III), and categorization of source localization algorithms by application (Section IV).

II. ALGORITHMS FOR SOURCE LOCALIZATION

A. TOA and TDOA-based algorithms with LOS

The general 3D range equations for source localization using TOA and TDOA are

\[
\text{TOA} : \\
\end{aligned}
\[
\text{TDOA} : \\
\end{aligned}
\]

where \( s \) is the signal propagation velocity, \( t_i \) is the signal traveling time from the source to sensor \( i \), and \( \Delta t_{ij} \) indicates the time difference between travel times \( t_i \) and \( t_j \). The terms \( x_i, y_i, z_i \) represent the position of sensor \( i \), and \( N \) is the number of sensors (\( N \geq 4 \)). The terms \( x, y, z \) are the coordinates for the position of the source that are to be determined. Nonlinearity of the source localization problem is introduced into the algorithm by the square-root terms (distance formula) in Equations (2). These make estimation of a source’s location potentially complex and expensive.

To obtain the measurements of TOAs and TDOAs, a group of sensors first receives signals emitted from the source. The sources and receivers must be time-synchronized to ensure precision of the arrival times. In a passive system, the TDOA measurement can be made by cross-correlating the signals received at two different sensors. Alternatively, TOA measurements can be gathered at the sensors and converted into TDOA measurements by calculating the differences.

In geometric positioning, TOAs are then converted to range estimates by multiplying the propagation speed of light or sound in the appropriate medium. For each TOA, if accurately measured and assumed to be free of noise, the range estimate to the source results in a circular locus of possible source positions with the receiver at the center. This is commonly called the circular line of position (LOP). In a 2D space, at least three such LOPs are needed to estimate the source’s location. The position of the source is at a single point where all three LOPs intersect (Figure 1, solid line). In practical situations, TOAs have measurement errors and the circular LOPs may not have a unique intersection point. The trilateration will yield multiple intersections bounded by the errors in the TOA measurements (Figure 1, dashed line). The estimate of source position then is obtained by solving for an optimal solution. In a 3D space for TOAs measured without error, the source location locus for each TOA is the surface of a sphere. Three TOA sphere surfaces intersect at two points, so a fourth TOA is needed to determine the position of the source. TOAs can also be used to estimate TDOAs (Equation (2)), which are source range-difference estimates. In the 2D case, the possible location of a source for each TDOA is given by a hyperbolic LOP in which the focal points of the hyperbola are the positions of the two receivers used in the TDOA computation. At least two hyperbolas (Figure 2, solid line) formed using two TDOAs computed...
from TOAs for three receivers are needed to find the intersections of the two hyperbolas, the potential locations of the signal source. Because TOAs and therefore TDOAs have measurement errors, the location of a source may be estimated by propagating the errors through the computation and estimating the source location along with the errors in the estimates (Figure 2, dashed line). In the 3D case, a hyperboloid is defined by each TDOA, and at least three TDOAs need to intersect at a unique point to identify a source location. Intersection of LOPs, a geometric construct, is the most basic and intuitive method for source position estimation. Starting from this idea, numerous methods have been developed, validated, and published in the literature. The most used TOA- and TDOA-based source-localization algorithms when LOS between receivers and sources exist are summarized in Table III.

Direct solutions may be used when TOA measurement errors are not considered. The source position can be directly calculated from different sets of geometric equations (Schmidt, 1972; Fang, 1990; and Caffery, 2000). The computational complexity of this approach to source localization is low, but frequently does not result in a source location solution when TOA measurement errors exist. Furthermore, when additional LOPs are available in an ODS, the source position needs to be estimated by averaging all the possible source locations (LOP intersections). A linear least-squares (LS) approach is an alternative source location estimation technique (Caffery, 2000) that can provide a global solution using simple computational steps for situations when only the minimum number of sensors (MRS) is available. It can also be used in the ODS case. However, it is a suboptimal technique because, by assuming sufficiently small measurement errors, the source location estimate accuracy is not high. As a result, other types of approaches have been developed to improve source location estimate accuracy when there is error in TOA measurements. Measurement errors are usually assumed to be independent, zero-mean Gaussian variables with the same variance for all receivers in an LOS environment. Weighted LS estimators are computed by introducing a weight matrix into the LS cost function. A favorite LS weight matrix is the inverse of the covariance matrix of TOA measurement errors (Chan et al., 2006a).

<table>
<thead>
<tr>
<th>Localization measurement</th>
<th>Characteristics</th>
<th>Advantages</th>
<th>Disadvantages</th>
<th>Usage and applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOA</td>
<td>Performing direct ranging from the relationship of signal traveling time, speed, and distance</td>
<td>High accuracy&lt;sup&gt;a&lt;/sup&gt;</td>
<td>LOS is normally assumed</td>
<td>More common in cellular networks</td>
</tr>
<tr>
<td>TDOA</td>
<td>Presenting difference of TOAs from a pair of sensors</td>
<td>High accuracy&lt;sup&gt;a&lt;/sup&gt;</td>
<td>LOS is normally assumed</td>
<td>More common in wireless sensor networks</td>
</tr>
<tr>
<td>AOA</td>
<td>Intersecting the direction lines obtained from the angles measured at the sensors.</td>
<td>Only at least two receivers are needed</td>
<td>Smart antennas are needed, relatively larger and expensive hardware.</td>
<td>Common in radar scenarios.</td>
</tr>
<tr>
<td></td>
<td>Computation requires information of each sensor’s orientation</td>
<td>Time synchronization is not required</td>
<td>LOS is normally assumed</td>
<td>More appropriate for sensors rather than transmitters due to large size. Or source size has to be able to carry an antenna array</td>
</tr>
<tr>
<td>RSS</td>
<td>Comparing RSS measurements from source to each sensor with a propagation model to estimate distance</td>
<td>Time synchronization is not required</td>
<td>Low accuracy</td>
<td>Typically used in applications that do not require accurate ranging</td>
</tr>
<tr>
<td>Hybrid</td>
<td>TOA/RSS and TDOA/RSS</td>
<td>Relatively simple hardware requirement</td>
<td>Utilized for NLOS condition and indoor environment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOA/AOA and TDOA/AOA</td>
<td>Better performance when source is in proximity to sensors; allows for single sensor localization</td>
<td>Utilized for NLOS condition and indoor environment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TDOA/FDOA</td>
<td>Complementary to TDOA for estimating source position and velocity</td>
<td>Common in moving source (mobile) localization</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOA/TDOA</td>
<td>Data fusion</td>
<td>Utilized for NLOS condition</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Accuracy is dependent on the signal bandwidth.
The general LS cost function for TOA measurements and TDOA measurements are

\[ J_{\text{TOA}} = \sum_{i=1}^{N} (t_i s - \|\tilde{x} - x_i\|)^2, \tag{3} \]

\[ J_{\text{TDOA}} = \sum_{i=2}^{N} \frac{(t_i - t_1)s - \|\tilde{x} - x_i\| + \|\tilde{x} - x_1\|)^2}{\sigma_i^2}, \tag{4} \]

where \( t_i \) is the measured TOA at sensor \( i \) with or without measurement errors, \( x_i \) is the coordinate vector of sensor \( i \), \( \sigma_i^2 \) is the variance of TOA measurements at sensor \( i \), and \( \sigma_{d,i}^2 \) is the variance of TDOA measurement at sensor \( i \).

The objective of the algorithm is to estimate the source coordinate vector \( \tilde{x} \) that minimizes the cost function \( J \). Iterative nonlinear minimization is required for an optimal solution. A common solution technique is to create an iterative algorithm based on an initial position estimate obtained using the Gauss-Newton method, the steepest descent method, or the combined Levenberg-Marquardt method, which typically have high computation requirements. A good initial source position estimate also is needed to find the global minimum. Therefore, a Taylor series (TS) is often used to linearize the nonlinear equations by updating a LS solution (Foy, 1976 and Torrieri, 1984). The Taylor series methods refine the initial guess by iterating a procedure determining location estimation errors. Compared with LS methods that ignore measurement errors, TS methods can provide accurate location estimation at reasonable noise levels. However, they still require a fairly accurate initial estimate of source location, and convergence on a source location estimate is not guaranteed. The ML approach estimates source position by minimizing the cost function of the probability density function of measurements (Ziskind and Wax, 1988). This algorithm is similar to a weighted nonlinear LS approach when measurement errors are zero-mean Gaussian distributed (Chan et al., 2006a). Li et al. (2014) improved on an algorithm published by Chan et al. (2006b) using an efficient approximate ML algorithm that included coupling with bad-measurement filters for 3D source localization.

To avoid iterative algorithms, two-stage, closed-form LS estimators have been extensively developed for ML approximation (Friedlander, 1987; Schau and Robinson, 1987; Smith and Abel, 1987a; Smith and Abel, 1987b; Chan and Ho, 1994; Brandstein and Silverman, 1997; Huang et al., 2001; and Cheung et al., 2004). These LS solutions can provide good initialization for iterative estimators, which converge with less computational effort to a source position estimate with higher accuracy (Smith and Abel, 1987a and Chan et al., 2006a). Some researchers have compared the performance of algorithms (Yu and Oppermann, 2004; Shen et al., 2008; Gezici et al., 2008; and So, 2011). An LS methodology that used squared range or squared range-difference measurements (Beck et al., 2008) is an attractive approach in that the method performs very well even at high noise levels.

B. Algorithms dealing with specific scenarios

Because real environments are complex and the geometries of sources and sensors can be quite variable, algorithms have been developed for special cases. For instance, estimation of the location of a source under NLOS conditions is one of the main challenges in localization. Güvenç and Chong (2009) presented an overview of different TOA NLOS localization algorithms with varying levels of computational complexity and prior information. Early researchers considered treatment methods that identify the NLOS sensors in an array using TOA measurements, then estimate the location of the source.
## TABLE III.
Summary of TOA and TDOA-based, source-localization algorithms in LOS condition.

<table>
<thead>
<tr>
<th>Author and year</th>
<th>Dimension and measurement</th>
<th>Algorithm</th>
<th>Advantage and disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Schmidt, 1972)</td>
<td>2D/3D, TDOA</td>
<td>LOCA: Location on the conic axis, an alternative geometry to the hyperbolic intersection principle, also known as plane intersection (PX) method</td>
<td>The sensors appear on the conic rather than at foci and thus the source locations appear at foci rather than on a hyperbola</td>
</tr>
<tr>
<td>(Foy, 1976)</td>
<td>2D, TOA/TDOA/AOA</td>
<td>Taylor-series, an iterative Gauss-Newton method, gives LS solution</td>
<td>Requires an initial guess, not a simple start in application&lt;br&gt;Convergence is not proved&lt;br&gt;Is computationally expensive&lt;br&gt;Useful in solving multiple-measurement, mixed-mode problems</td>
</tr>
<tr>
<td>(Knapp and Carter, 1976)</td>
<td>TOA/TDOA</td>
<td>ML estimator, the generalized cross-correlation method</td>
<td>The most widely used method to obtain time-delay estimates, a landmark paper</td>
</tr>
<tr>
<td>(Torrieri, 1984)</td>
<td>2D, TOA/TDOA/AOA</td>
<td>Taylor-series method, linearizing the hyperbolic location equations in an iterative algorithm</td>
<td>Similar disadvantages to Foy (1976)</td>
</tr>
<tr>
<td>(Friedlander, 1987)</td>
<td>3D, TDOA</td>
<td>Weighted LS method</td>
<td>Localizations from MRS and ODS are both considered&lt;br&gt;Also derived a linearization algorithm to estimate source velocity from TDOA/FDOA</td>
</tr>
<tr>
<td>(Schau and Robinson, 1987)</td>
<td>3D, TDOA</td>
<td>Spherical intersection (SX) method</td>
<td>Only presented solution for MRS&lt;br&gt;Requires a priori solution for the source range</td>
</tr>
<tr>
<td>(Smith and Abel, 1987a)</td>
<td>3D, TDOA</td>
<td>Spherical interpolation (SI) method: A closed-form two-step LS method, linear LS &quot;equation-error&quot; minimization</td>
<td>Closely related to ML solution for Gaussian TDOA measurement errors&lt;br&gt;Non-iterative algorithm</td>
</tr>
<tr>
<td>(Smith and Abel, 1987b)</td>
<td>3D, TDOA</td>
<td>SI method: An intermediate term introduced linearized the nonlinear hyperbolic equations</td>
<td>It has one-order-of-magnitude greater noise immunity than the SX method&lt;br&gt;Has consistently lower variance and slightly higher bias than the PX method&lt;br&gt;Source range is independent of the location coordinates, exhibiting worse performance with larger noise levels</td>
</tr>
<tr>
<td>(Fang, 1990)</td>
<td>3D, TDOA</td>
<td>An exact solution to the hyperbolic TDOA equations, when the number of TDOAs is equal to the number of coordinates of source</td>
<td>Clear and simple solutions</td>
</tr>
<tr>
<td>(Chan and Ho, 1994)</td>
<td>2D, TDOA</td>
<td>Approximation of ML estimator, improved from SI method&lt;br&gt;Two-stage weighted LS method: An unconstrained least-squares solution is obtained first and then a second LS estimator utilized the constraint between source coordinates and the intermediate variable to refine the position coordinates</td>
<td>It is non-iterative, explicit solution only when the TDOA measurement errors are small (at high SNR)&lt;br&gt;Performs significantly better than SI method, particularly when the number of sensors is small&lt;br&gt;Computational burden is similar to SI method but much lower than Taylor-series methods (Torrieri, 1984)&lt;br&gt;Need priori knowledge of the second-order statistics of the TDOA measurement errors</td>
</tr>
<tr>
<td>(Brandstein and Silverman, 1997)</td>
<td>3D, TDOA</td>
<td>Linear intersection (LI) method: Calculate a number of potential source locations from the points of closest intersection for all pairs of bearing lines and use a weighted average of these locations for a final estimate</td>
<td>Localization in 2D can be performed with a 2D sensor system&lt;br&gt;Performs better than SI method for moderate and large noise levels</td>
</tr>
</tbody>
</table>
TABLE III. (Continued.)

<table>
<thead>
<tr>
<th>Author and year</th>
<th>Dimension and measurement</th>
<th>Algorithm</th>
<th>Advantage and disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Caffery, 2000)</td>
<td>2D, TOA</td>
<td>A new geometric interpretation replaced circular LOPs with linear LOPs to determine the source position, by averaging the intersections of linear LOPs</td>
<td>The linear LS algorithm was significantly better than Taylor-series LS algorithm when a large bias was present. Widely used for NLOS environments</td>
</tr>
<tr>
<td>(Huang et al., 2001)</td>
<td>3D, TDOA</td>
<td>A linear-correction LS approach incorporates the relation between source coordinates and the intermediate variable from SI method explicitly based on Lagrange multipliers. Use additive measurement error model</td>
<td>Can be easily implemented in a real-time system. Make no assumptions about the noise covariance. More efficient than SI method</td>
</tr>
<tr>
<td>(Cheung et al., 2004)</td>
<td>2D, TOA</td>
<td>A constrained weighted LS (CWLS) algorithm extends the SI method using the TOA measurements</td>
<td>For sufficiently small measurement errors. Has performance optimality and capability of extension to hybrid measurement cases (Cheung et al., 2006)</td>
</tr>
<tr>
<td>(Chan et al., 2006a)</td>
<td>2D, TOA/TDOA</td>
<td>A close-form approximate solution to the ML equation. A weighted LS solution is used as initial guess to calculate the weighting matrix in exact ML estimator. The approximate ML solution is obtained by updating the solution iteratively</td>
<td>Insensitive to geometry, thus it is superior to Chan and Ho (1994) and Caffery (2000). Gives an exact ML estimate when three sensors on a straight line</td>
</tr>
</tbody>
</table>

without using the NLOS sensors, or reduce the error in position estimates by weighting the NLOS less (Wylie and Holtzman, 1996; Chen, 1999; and Chan et al., 2006b). This approach is effective when a small number of the distributed sensors are NLOS, but become computationally intensive when there are many distributed sensors and may not be possible in situations where the signal source is moving. Because NLOS-caused errors are always positive, this constraint can be imposed on the search for a position estimate (Wang et al., 2003; Venkatraman et al., 2004; and Cong and Zhuang, 2005), but normally, prior knowledge of NLOS error statistics is required to correct a LOS estimate. The procedure for estimating a source location when some distributed sensors are NLOS is more complicated using TDOAs because the NLOS error for a reference sensor will be transferred to all TDOAs computed using that sensor. Additional research is needed to learn how to mitigate NLOS errors effectively and efficiently under real world conditions with moving sources and large receiving networks.

Proficiency in estimating the locations of fixed sources is becoming less important as interest in detecting, locating, and tracking mobile sources has increased. Frequency difference of arrival (FDOA) measurements with independent sources of errors are combined with TDOAs to immediately estimate the location and velocity of a source (Ho and Chan, 1997; Ho and Xu, 2004; and Anderson et al., 2005). Kalman filter (Leonard and Durrant-Whyte, 1991) or Monte Carlo approaches (Sheng et al., 2005) can be integrated into tracking algorithms for mobile source localization to improve their performance. These additions can detect discontinuities from continuously estimated positions and smooth the source’s trajectory by replacing the position estimate outliers with predicted or refined estimates. Kalman filters, which are optimal recursive Bayesian estimators for linear Gaussian problems, are the best-known filters. Extended (Welch and Bishop, 2006) and unsecented Kalman filters (Julier and Uhlmann, 1997) are examples of nonlinear filtering by linearization. Particle filters are the most general class of filters for nonlinear and non-Gaussian problems (i.e., sequential Monte Carlo) (Gordon et al., 1993; Kitagawa, 1996; and Ristic et al., 2004). Computational cost is a major disadvantage of Monte Carlo estimators. The novel challenge of estimating the locations of mobile sources using mobile sensors is now being investigated (Ho and Xu, 2004 and Hu and Evans, 2004). Optimal design of a sensor system (i.e., network, array) is a challenging task that can significantly affect tracking accuracy.

III. PERFORMANCE ANALYSIS

A. Sources of error

3D localization and tracking requires at least four different sensors to form the necessary nonlinear localization equations. From Equation (2), the precision of the estimate of a source’s location can be estimated as a function of the errors in the measurements of sensor locations, TOA/TDOA, and signal velocity. Under NLOS conditions, an additional term for the distance bias caused by the blockage of the direct path between source and sensor should be included in the localization equations.

In many cases, accurate estimates of sensor locations can be obtained from careful field surveys and highly accurate GPS location estimates. In homogeneous media, the speed
of a signal can be accurately estimated by using known relationships between the characteristics of the transmitted signal (e.g., sound or radio) and of the medium through which the signal will propagate (e.g., air or water). For instance, a fifth-order polynomial can accurately describe the dependence of signal speed in water on temperature (Marczak, 1997). In heterogeneous media, where the signal speed differs from path to path, “isodiachron” algorithms can improve the accuracy of location estimates (Spiesberger, 2004).

However, the errors in TOA and TDOA measurements can be quite complicated, depending on multiple factors (Mao et al., 2007), such as SNR, integration time, signal bandwidth, multipath propagation, and possible NLOS propagation. In practical implementations, hardware limitations and transmission channel degradation may become the dominant sources of error (Krizman et al., 1997 and Caffery and Stuber, 1998). Considering hardware limitations, TDOA measurements require highly precise synchronization between sensors, whereas TOA measurements require accurate synchronization between a transmitter and sensor. Such hardware requirements can greatly increase the complexity and cost of localization systems. Minimizing the errors in time of arrival and other time-based measurements is the greatest challenge in many source localization studies because of the large uncertainties in such measurements. For example, ultra-wide-band signals are used for wireless positioning (Gezici et al., 2005) because of their time domain high-resolution capability.

B. Theoretical analysis

Benchmarks are needed to assess the performance of localization estimators. The Cramér-Rao Lower Bound (CRLB), which gives the minimum variance of unbiased estimators, is widely used as a measure of the precision attainable for parameter estimates from a given set of observations (Van Trees et al., 1968 and Kay, 1993). The CRLB was compared with the mean square errors of different localization algorithms under low SNR conditions (Gustafsson and Gunnarsson, 2005 and Macagnano et al., 2012). Using TOA and TDOA measurements, So (2011) used the corresponding Fisher information matrix with zero-mean Gaussian distributed measurement errors to compute CRLB in LOS scenarios. An example of 2D localization for TOA measurements is

\[
CRLB(x) = [I^{-1}(x)]_{11}^{-1} + [I^{-1}(x)]_{22}^{-1},
\]

\[
I(x) = \begin{bmatrix}
\sum_{i=1}^{N} \frac{(x-x_i)^2}{\sigma_d^2 d_i^2} & \sum_{i=1}^{N} \frac{(x-x_i)(y-y_i)}{\sigma_d^2 d_i^2} \\
\sum_{i=1}^{N} \frac{(x-x_i)(y-y_i)}{\sigma_d^2 d_i^2} & \sum_{i=1}^{N} \frac{(y-y_i)^2}{\sigma_d^2 d_i^2}
\end{bmatrix},
\]

\[
d_i = |x-x_i|, \quad x = [x, y]^T.
\]

In NLOS scenarios, the CRLB depends on LOS signals, assuming that the NLOS sensors can be accurately identified (Güvenç and Chong, 2009). Qi and Kobayashi (2002) derived an explicit formulation of the CRLB for NLOS geolocation. CRLBs for the TOA and TDOA based source localization estimates are determined by (1) the positions of the sensors, (2) the positions of the source, and (3) variances of measurement noise, \(\sigma^2\). Independent of \(\sigma^2\), the CRLB indicates that achievable localization accuracy is related to the geometry of the distributed sensors relative to the source locations.

Another approach to estimate source position accuracy is to directly compute position errors using distance equations, which does not require expensive Monte-Carlo simulations. This type of theoretical error analysis was performed on 2D MRS systems and compared with field measured positions (Smith et al., 1998). Wahlberg et al. (2001) extended the mathematical approach of this method (Watkins and Schevil, 1971) to formulate a linear error propagation model. The position accuracy for 2D and 3D tracking for both MRS and ODS sensor systems was estimated as a function of TDOA errors, sound velocity errors, and sensor position errors, and they concluded sensor position errors had a large impact on source position estimate accuracy. Later, Ehrenberg and Steig (2002) presented a derivation of an expression that showed the dependence of source position error on TOA and sound velocity estimate errors, which are independent of the algorithm used to determine the source position estimate. To minimize errors in source position estimates, in addition to optimization of signal processing, an optimal distributed sensor geometry is required to decrease the sensitivity of estimated source positions to TOA/TDOA measurement errors (Ehrenberg and Steig, 2003).

IV. APPLICATIONS OF SOURCE LOCALIZATION

A. Radar and sonar

Radar and sonar source localization systems have been used to detect, localize, and track signal sources for decades (Altes, 1979; Krim and Viberg, 1996; and Le Chevalier, 2002). Radar and sonar have been widely used for civilian, military, and scientific purposes, and the range of applications is large (Minkoff, 1992). The applications of sonar and radar systems are separated into active (Bekkerman and Tabrikian, 2006) and passive system (Carter, 1981) problems. Active sonars and radars transmit signals that are reflected back from targets (i.e., an echo), whereas passive sonars and radars do not transmit and only receive signals transmitted by sources (Stergiopoulos, 2000). There are many similarities between radar (using radio waves) and sonar (using sound) signal processing, and developments for both types of systems have contributed significantly to the solutions to underwater localization problems (Vaccaro, 1998).

B. Wireless sensor networks

Determination of the location of nodes in wireless sensor networks accurately and at low cost is very important (Patwari et al., 2005 and Mao et al., 2007). Knowledge of the precise location of wireless network sensors is fundamental for a wide range of applications (i.e., event detection and tracking), such as disaster relief operations (e.g., wildfire detection), environmental monitoring, biodiversity mapping, and precision agriculture (Karl and Willig, 2007). Accuracy and precision are the most important characteristics for a WSN localization system. Thus, TDOA methods are more suitable and commonly used.
for WSNs, compared with the low-cost but less accurate RSS method (Boukerche et al., 2007).

Numerous methods have been developed to reduce the location estimation errors for WSN nodes, thereby enhancing the event detection and tracking functions of WSN. Optimization of the design of sensor networks for specific applications is one of the most important components of source localization systems. The design of the sensor network determines the applicability of localization algorithms and thus has been investigated for years (Sohrabi et al., 2000; Langendoen and Reijers, 2003; Römer and Mattern, 2004; Al-Karaki and Kamal, 2004; and Wang, 2008). To improve source localization, different types of signals (e.g., acoustic sound, ultrasound, or radio) transmitted by sources may result in different estimation errors (Boukerche et al., 2007). Whitehouse (2002) tested an acoustic source tracking system and observed source location estimation errors of approximately 23 cm. In other experiments computing the distance of radio and ultrasound signals using various network design scenarios (Savvides et al., 2001), the errors were as low as 2–3 cm. Novel computational techniques are also proving helpful in overcoming specific localization problems in WSNs (Kulkarni et al., 2011). For example, stochastic particle swarm optimization was recognized as an efficient tool to use to solve local-minimum problems for localization and tracking in mobile WSN environments (Gopakumar and Jacob, 2008).

C. Global positioning system

GPS provides coded satellite signals that can be processed to compute a receiver’s three dimensional position and velocity by using four or more GPS satellites (Kaplan and Hegarty, 2005). The global GPS system has benefited civil, commercial, and military users worldwide (Hofmann-Wellenhof et al., 2013) in wide-ranging applications such as ground surveying, land vehicle navigation and tracking, marine navigation, air traffic control, aircraft landing, general aviation, and geodesy (Parkinson, 1996). A variety of error sources are encountered using the GPS system. Among these are selective availability, clock and ephemeris errors, ionospheric delays, tropospheric delays, multipath, signal noise, receiver errors, poor satellite coverage, and satellite geometry (Dana, 1997; Kiefer and Lillesand, 2004; and Misra and Enge, 2006). The accuracy of GPS depends on receiver location and the presence of obstructions that may block satellite signals. With technical improvements to GPS receivers, it is possible to measure a location on Earth at high frequency (e.g., 5 Hz) at a centimeter level of precision using the phase differential positioning method for timing measurements (Schutz and Herren, 2000).

D. Underwater localization of animals

1. Fisheries

Telemetry systems are widely used to track fish to observe their behavior and obtain other information such as seasonal changes (e.g., preferences for water depths or temperatures), foraging behavior, habitat utilization, home range size, spawning behavior, site fidelity, and mortality (Priede et al., 1990; McCalfe and Arnold, 1997; Roussel et al., 2000; Meyer et al., 2000; Cunjak et al., 2005; and Skalski et al., 2001). Because many fish of interest are small, methods for implantation of sources within their bodies or external attachment are an important topic of research. The transmitter and the attachment method should be designed to have minimal effect on the fish after they are released (Deng et al., 2012).

An important area in the study of fish is the migration and passage behavior of fish through hydroelectric facilities (Evans and Johnston, 1980; Webb, 1975; Northcote, 1998; Cáda, 2001; and Schilt, 2007). Transmitters with frequency and pulse codes that permit fish to be individually identified can provide detailed information on both the large- and small-scale behavior of fish (Berman and Quinn, 1991). A recent example is the Juvenile Salmon Acoustic Telemetry System that is being used to monitor the survival and behavior of juvenile salmonids migrating downstream through eight large hydroelectric facilities within the Federal Columbia River Power System to the Pacific Ocean (McMichael et al., 2010; Weiland et al., 2011; Deng et al., 2011; and Deng et al., 2015).

2. Acoustic tracking of marine animals

It is well known that marine mammals use sound (calls) passively and actively to communicate. They use calls for foraging, predator avoidance, navigation, and environment sensing. Biologists have used acoustic tags (transmitters) to track the movement of marine animals over small and large scales (Heupel et al., 2006). Although the acceptable error in position accuracy varies with the objectives of individual behavioral studies on marine animals (Janik et al., 2000), these powerful tools have provided insights into the behavior of marine animals and have opened new opportunities for studying the ways they interact with the environment (Johnson et al., 2009).

New tagging techniques, analytical methods, and experimental designs are needed to match research objectives with acoustic tag performance in marine environments. An array of four hydrophones arranged in a symmetrical star configuration (phased array) was used to measure TDOAs of the echolocation signals of spotted dolphins (Au and Herzing, 2003) and killer whales (Au et al., 2004) to estimate their distances from the array. Brousseau et al. combined acoustic and radio transmitters to study horseshoe crab spawning behavior in subtidal habitats.

Approaches using TDOA measurements have been the most common methods used to monitor movement of animals in the ocean (Wartzok et al., 1992; Stafford et al., 1998; Holland et al., 1999; Thode, 2004; Morrissey et al., 2006; Muanke and Niezrecki, 2007; and Deng et al., 2013) and to estimate their population densities (Muanke and Niezrecki, 2007 and Marques et al., 2009).

E. Other applications

Electronically steerable arrays of microphones that apply TDOA technologies are used in a variety of ways in speech data acquisition systems (Brandstein and Silverman, 1997),...
Microphone arrays are capable of automatic detection, localization, and tracking of active “talkers” in an enclosed environment in the presence of reverberation (DiBiase et al., 2001 and Ma et al., 2006).

The use of cellular mobile systems to position mobile stations will play a fundamental role in future wireless communication networks (Caffery, 2006). Applications of wireless localization using code division multiple access cellular networks include E-911, fraud detection, cellular system design and resource management, fleet management, and intelligent transportation systems (Caffery and Stuber, 1998). These position- ing algorithms utilize TDOA approaches (Zhu and Zhu, 2001 and Mensing and Plass, 2006) or hybrid TDOA/AOA (Jami et al., 1999 and Cong and Zhuang, 2002).

V. CONCLUSIONS

This article has reviewed localization algorithms that apply TOA and TDOA in transmitter and receiver technologies for a wide range of applications. Compared with other signal source localization approaches, TOA and TDOA are both appropriate for applications that require high accuracy. We have summarized some of the most popular 2D and 3D location estimation algorithms and compared them for accuracy and computational efficiency. Many factors can influence the performance of localization algorithms in specific applications. Among these are the design of distributed sensor or mobile networks; sensor position estimation accuracy (e.g., accurate surveying of sensor positions, node locations, mobility in network, etc.); consideration of environmental conditions; uncertainties in propagation (e.g., NLOS, multipath, sound speed variation, etc.); device limitations (e.g., synchronization, channel structure, etc.); and evaluation and validation of localization algorithms. Despite considerable technological development and improvements in hardware and software technologies, researchers still face significant challenges in developing approaches for location estimation of signal sources that are economical and provide high levels of performance.

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